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In the framework of minimal supersymmetric model we examine the  $Z$ -peak constraints on the scenario of one light sbottom ( $2 \sim 5.5$  GeV) and light gluino ( $12 \sim 16$  GeV), which has been successfully used to explain the excess of bottom quark production in hadron collision. Such a scenario is found to be severely constrained by LEP  $Z$ -peak observables, especially by  $R_b$ , due to the large effect of gluino-sbottom loops. To account for the  $R_b$  data in this scenario, the other mass eigenstate of sbottom, i.e., the heavier one, must be lighter than 125 (195) GeV at  $2\sigma$  ( $3\sigma$ ) level, which, however, is disfavored by LEP II experiments.

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**Introduction** Although the standard model (SM) has been successful phenomenologically, it is generally believed to be an effective theory valid at the electroweak scale and some new physics must exist beyond the SM. This belief was seemingly corroborated by some experiments, such as the recent measurement of muon  $g - 2$  [1] and the evidence of neutrino oscillations [2]. Among various speculations of new physics theories, the minimal supersymmetric model (MSSM) is arguably a promising candidate and has been intensively studied in the past decades.

The unobservation of any sparticles from direct experimental searches suggested heavy masses for sparticle spectrum. However, there have been a lot of analysis [3] which argue that a very light sbottom and light gluino (with mass of a few GeV) may have escaped from the direct experimental searches. A recent analysis [4] showed that a light sbottom ( $\tilde{b}_2$ ) with mass comparable with bottom quark is still allowed by electroweak precision data if its coupling to  $Z$  boson is small enough. A study by Berger *et al* [5] found that the scenario of MSSM with one light sbottom ( $2 \sim 5.5$  GeV) and a light gluino ( $12 \sim 16$  GeV) can successfully provide an explanation for the long-standing puzzle that the measured cross section of bottom quark production at hadron collider exceeds the QCD prediction by about a factor of 2 [6]. They also argued that such a scenario is consistent with all experimental constraints on the masses and couplings of sparticles.

We note that the previous examinations [4,5] on  $Z$ -peak constraints focus on the direct production of a light sbottom followed by its decay similar to the bottom

quark. Then by fine-tuning the mixing of left- and right-handed sbottoms, the coupling of  $Z$ -boson to the lighter mass eigenstate of sbottom ( $\tilde{b}_2$ ) can be sufficiently small so as to avoid the  $Z$ -peak constraints. It is noticeable that when sbottom  $\tilde{b}_2$  and gluino are both light, as was used to explain the excess of bottom quark production in hadron collision [5], gluino-sbottom loops may cause large effects in  $Zb\bar{b}$  coupling<sup>1</sup>. Therefore, in such a scenario, it is important to reexamine the loop contributions to  $Zb\bar{b}$  coupling and further, the  $Z$ -peak constraints. This is the aim of this letter. Through explicit calculations, we do find that gluino-sbottom loops comprising of sbottoms and a light gluino cause large effects on  $Z$ -peak observables. To account for the  $R_b$  data, subtle cancellation between  $\tilde{b}_2$  loops and  $\tilde{b}_1$  loops is needed, which can be realized by requiring the mass splitting between two sbottoms not to be too large. Numerical results show that for  $\tilde{b}_2$  with mass of  $2 \sim 5.5$  GeV,  $\tilde{b}_1$  must be lighter than 125 GeV and 195 GeV at  $2\sigma$  and  $3\sigma$  level, respectively.

**Calculations** We start the calculations by writing down the sbottom mass-square matrix [8]

$$M_{\tilde{b}}^2 = M^2 = \begin{pmatrix} M_{\tilde{b}_{LL}}^2 & M_{\tilde{b}_{LR}}^{2\dagger} \\ M_{\tilde{b}_{LR}}^2 & M_{\tilde{b}_{RR}}^2 \end{pmatrix}, \quad (1)$$

where  $M_{\tilde{b}_{LL}}^2 = M_{\tilde{Q}}^2 + m_b^2 - m_Z^2(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W)\cos(2\beta)$ ,  $M_{\tilde{b}_{RR}}^2 = M_{\tilde{D}}^2 + m_b^2 - \frac{1}{3}m_Z^2\sin^2\theta_W\cos(2\beta)$ , and  $M_{\tilde{b}_{LR}}^2 = m_b(A_b - \mu\tan\beta)$ . Here  $M_{\tilde{Q}}^2$  and  $M_{\tilde{D}}^2$  are soft-breaking mass terms for left-handed squark doublets  $\tilde{Q}$  and right-handed down squarks, respectively.  $A_b$  is the coefficient of the trilinear term  $H_1\tilde{Q}\tilde{D}$  in soft-breaking terms and  $\tan\beta = v_2/v_1$  is the ratio of the vacuum expectation values of the two Higgs doublets. By diagonalizing the sbottom mass-square matrix, one obtains the the physical mass eigenstates  $\tilde{b}_{1,2}$

$$\begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{b}_L \\ \tilde{b}_R \end{pmatrix}. \quad (2)$$

where  $\theta$  is the mixing angel of sbottoms. In our following analyses we take the sbottom masses and the mixing an-

<sup>1</sup>Previous calculations of SUSY loop effects on  $Zb\bar{b}$  coupling focused on rather heavy squarks and gluino and thus obtained very small effects [7].

gel as free parameters since they are independent of each other and determined by SUSY parameters  $M_{b_{LL}}^2$ ,  $M_{b_{RR}}^2$  and  $M_{b_{LR}}^2$ .

The coupling of  $Z$ -boson to sbottoms is given by

$$V^\mu(Z\tilde{b}_i\tilde{b}_j^*) = ieO_{ij}(p_1 + p_2)^\mu, \quad (3)$$

where  $p_{1,2}^\mu$  are the momentum of  $\tilde{b}_i$  and  $\tilde{b}_j$ , respectively;  $O_{ij}$  are defined by

$$\begin{aligned} O_{11} &= v_b + a_b \cos 2\theta, \\ O_{22} &= v_b - a_b \cos 2\theta, \\ O_{12} &= O_{21} = -a_b \sin 2\theta, \end{aligned} \quad (4)$$

where  $v_b = 1/(4 \sin \theta_W \cos \theta_W)(1 - \frac{4}{3} \sin^2 \theta_W)$  and  $a_b = 1/(4 \sin \theta_W \cos \theta_W)$  are the vector and axial vector couplings of  $Zb\bar{b}$ , respectively.

Apparently, a light sbottom  $\tilde{b}_2$  (a few GeV) can affect  $Z$ -peak observables in two ways: (1) the direct pair production of  $\tilde{b}_2$  through  $Z\tilde{b}_2\tilde{b}_2^*$  coupling, as discussed in [4]; (2) the loop effects of  $\tilde{b}_2$ . If gluino is also light ( $12 \sim 16$  GeV), then the loop effects are mainly from gluino-sbottom loops in  $Zb\bar{b}$  vertex, which comprise a light gluino  $\tilde{g}$  and sbottoms, as shown in Fig. 1. It should be noted that even if the direct pair production of  $\tilde{b}_2$  is avoided by tuning the mixing angle  $|\sin \theta| \simeq \sqrt{2/3} \sin \theta_W \simeq 0.38$  to set  $Z\tilde{b}_2\tilde{b}_2^*$  coupling to be zero ( $O_{22} \sim 0$ ),  $Z\tilde{b}_1\tilde{b}_1^*$  and  $Z\tilde{b}_1\tilde{b}_2^*$  couplings still exist and the irreducible loops shown in Fig. 1(b) make contributions. It should also be noted that the self-energy loops in Fig. 1(a) involve only SUSY QCD interactions, i.e., gluino-sbottom-bottom couplings, which are not affected by the zero  $Z\tilde{b}_2\tilde{b}_2^*$  coupling.

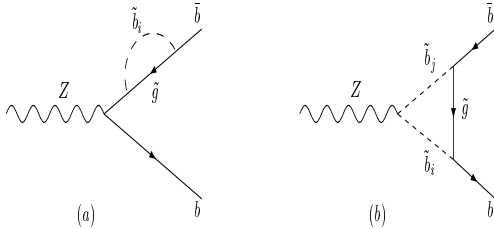


FIG. 1. The gluino one-loop diagrams for  $Zb\bar{b}$ .

Using dimensional regulation and adopting the on-shell renormalization scheme for the calculation of Fig. 1, we obtain the effective  $Zb\bar{b}$  vertex

$$\begin{aligned} V_\mu^{eff}(Zb\bar{b}) &= ie \left\{ \gamma_\mu (v_b - a_b \gamma_5) + \frac{\alpha_s}{3\pi} [F_1 \gamma_\mu + F_2 \gamma_\mu \gamma_5 \right. \\ &\quad \left. + i F_3 \sigma_{\mu\nu} k^\nu + i F_4 \sigma_{\mu\nu} k^\nu \gamma_5] \right\}. \end{aligned} \quad (5)$$

Here  $F_i$  are form factors originated from loop corrections, given by

$$\begin{aligned} F_1 &= 2 \sum_{i,j=1}^2 O_{ij} \{ -A_{ij}^- m_b m_{\tilde{g}} (C_0(i,j) + C_{11}(i,j)) \\ &\quad A_{ij}^+ [m_b^2 (C_{11}(i,j) + C_{21}(i,j)) + C_{24}(i,j)] \} \\ &\quad + v_b \delta Z_V + a_b \delta Z_A, \end{aligned} \quad (6)$$

$$F_2 = 2 \sum_{i,j=1}^2 O_{ij} B_{ij}^+ C_{24}(i,j) - v_b \delta Z_A - a_b \delta Z_V, \quad (7)$$

$$\begin{aligned} F_3 &= - \sum_{i,j=1}^2 O_{i,j} \{ A_{ij}^+ m_b [C_{11}(i,j) + C_{21}(i,j)] \\ &\quad - m_{\tilde{g}} A_{ij}^- [C_0(i,j) + C_{11}(i,j)] \}, \end{aligned} \quad (8)$$

$$\begin{aligned} F_4 &= \sum_{i,j=1}^2 O_{i,j} \{ B_{ij}^+ m_b [2C_{12}(i,j) - C_{11}(i,j) - C_{21}(i,j) \\ &\quad + 2C_{23}(i,j)] + m_{\tilde{g}} B_{ij}^- [(C_0(i,j) + C_{11}(i,j))] \}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} \delta Z_V &= \sum_{i=1}^2 \left[ A_{ii}^+ \left( B_1(i) + 2m_b^2 \frac{\partial B_1(i)}{\partial p_1^2} \right) \right. \\ &\quad \left. - 2m_b m_{\tilde{g}} A_{ii}^- \frac{\partial B_0(i)}{\partial p_1^2} \right] \Big|_{p_1^2 = m_b^2}, \end{aligned} \quad (10)$$

$$\delta Z_V = - \sum_{i=1}^2 B_{ii}^+ B_1(i). \quad (11)$$

Here  $B_{0,1}(j) = B_{0,1}(-p_b, k, m_{\tilde{g}}, m_{\tilde{b}_j})$  and  $C_{nm}(i,j) = C_{nm}(-p_b, k, m_{\tilde{g}}, m_{\tilde{b}_i}, m_{\tilde{b}_j})$ , with  $p_b$  and  $k$  denoting the four-momentum of  $b$  quark and  $Z$  boson respectively, are the Feynman loop integral functions and their expressions can be found in [9]. Other constants appearing above are defined by

$$\begin{aligned} A_{ij}^\pm &= a_i a_j \pm b_i b_j, \quad B_{ij}^\pm = a_i b_j \pm a_j b_i, \\ a_{1,2} &= \frac{1}{\sqrt{2}} (\sin \theta \mp \cos \theta), \quad b_{1,2} = \frac{1}{\sqrt{2}} (\cos \theta \pm \sin \theta). \end{aligned} \quad (12)$$

Using above result, we have checked that all the ultraviolet divergencies in  $F_i$  are canceled, which results from the renormalizability of MSSM.

**Numerical results** Let's now evaluate the effects of the above corrections to  $Z$  peak observables. We start with  $R_b \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow hadrons)$ . From Eq. (5) we obtain the contribution to  $R_b$

$$\delta R_b = R_b^{SM} (1 - R_b^{SM}) \Delta_{\text{SUSY}}, \quad (13)$$

where

$$\begin{aligned} \Delta_{\text{SUSY}} &= \frac{2\alpha_s}{3\pi} \frac{1}{v_b^2 (3 - \beta^2) + 2a_b^2 \beta^2} [v_b (3 - \beta^2) \text{Re} F_1 \\ &\quad - 2a_b \beta^2 \text{Re} F_3 + 6m_b v_b \text{Re} F_4] \end{aligned} \quad (14)$$

with  $\beta = \sqrt{1 - 4m_b^2/m_Z^2}$ .

To obtain numerical results, we set input parameters as [10]

$$\begin{aligned} R_b^{exp} &= 0.21642 \pm 0.00065, & R_b^{SM} &= 0.21573 \pm 0.0002, \\ \sin^2 \theta_W &= 0.2312, & \alpha_s(m_Z) &= 0.1192, \\ m_Z &= 91.188, & m_b &= 4.75. \end{aligned}$$

We will vary  $m_{\tilde{g}}$  in the range  $12 \sim 16$  GeV and  $m_{\tilde{b}_2}$  in the range  $2 \sim 5.5$  GeV as was used in [5].

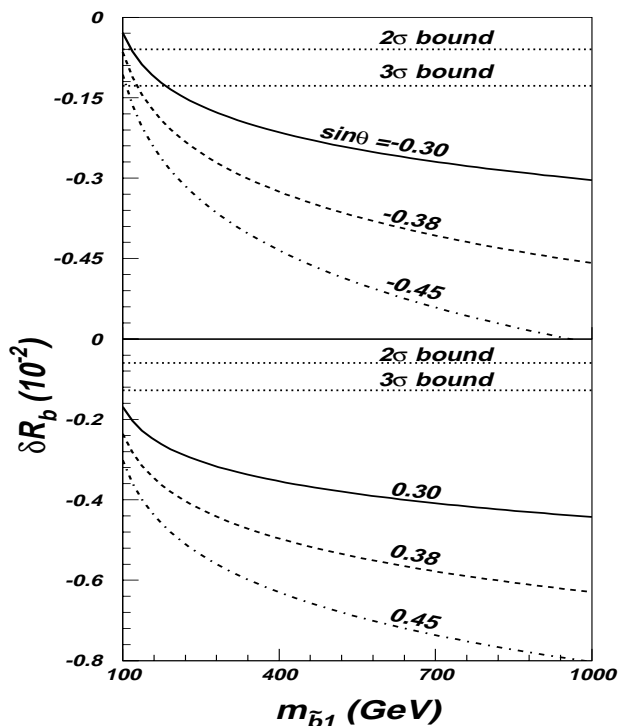


FIG. 2.  $\delta R_b$  as a function of  $m_{\tilde{b}_1}$  for  $m_{\tilde{b}_2} = 3.5$  GeV and  $m_{\tilde{g}} = 14$  GeV. The corresponding region above each horizontal line is allowed by LEP  $R_b$  data at  $2\sigma$  and  $3\sigma$  level, respectively.

For  $m_{\tilde{b}_2} = 3.5$  GeV and  $m_{\tilde{g}} = 14$  GeV, we present  $\delta R_b$  versus  $m_{\tilde{b}_1}$  in Fig. 2. In addition to  $\sin \theta = \pm 0.38$  which leads to zero  $Z\tilde{b}_2\tilde{b}_2^*$  coupling and hence avoids large rate of direct pair production of  $\tilde{b}_2$  [4], we also plotted the curves for  $\sin \theta = \pm 0.30$  and  $\pm 0.45$ . From the figure, one sees that the contributions to  $R_b$  are negative in all the parameter space we have investigated. One can also see that the positive  $\sin \theta$  gives larger contributions than negative one and as  $|\sin \theta|$  increases, the contributions become more sizable.

Comparing with the experimental bounds shown in Fig. 2, one learns that even in the favorable case of negative  $\sin \theta$ , the contribution to  $R_b$  is too large to be allowed at  $3\sigma$  level if  $m_{\tilde{b}_1} \geq 200$  GeV. Since the heavier sbottom has not been observed at LEP II, and it can in principle be produced in association with the lighter one, its mass should be larger than, conservatively, 200 GeV. So, we conclude that the scenario of one light sbottom and light

gluino faces severe challenge. As to the largeness of the gluino-sbottom loop contributions, two main reasons may account for it. One is the large splitting between  $m_{\tilde{b}_1}$  and  $m_{\tilde{b}_2}$ , which leads to a weak cancellation between  $\tilde{b}_1$  and  $\tilde{b}_2$  contributions; the other is the lightness of sbottom  $\tilde{b}_2$  and gluino, which induces large self-energy contributions. To check our understanding, we fix  $m_{\tilde{b}_1}$  and  $m_{\tilde{g}}$  but let  $m_{\tilde{b}_2}$  approaches to  $m_{\tilde{b}_1}$ . Then we do find large cancellation occurs between different diagrams.

We notice from Fig. 2 the intriguing feature that as  $m_{\tilde{b}_1}$  increases, the effects get more sizable. This can be understood as the weaker cancellation between  $\tilde{b}_1$  and  $\tilde{b}_2$  contributions when  $m_{\tilde{b}_1}$  increases. To further understand this behavior, we used approximate forms of B and C functions [9], and found that in the limit  $m_{\tilde{b}_1}^2 \gg m_Z^2 > m_{\tilde{b}_2, \tilde{g}}^2$ ,  $\delta R_b$  is roughly linear dependent on  $\ln(m_{\tilde{b}_1}^2/m_{\tilde{b}_2}^2)$  and thus increases as  $m_{\tilde{b}_1}^2/m_{\tilde{b}_2}^2$  gets larger. Of course, this feature does not mean that SUSY QCD is non-decoupling from the SM. To check the decoupling property of SUSY QCD, we let all relevant sparticles ( $\tilde{b}_1$ ,  $\tilde{b}_2$ ,  $\tilde{g}$ ) become heavy and found the contributions drop quickly to zero. Actually, even for a light  $\tilde{b}_2$ ,  $\delta R_b$  drops monotonously to zero when  $m_{\tilde{g}}$  get large, as shown in Fig. 3.

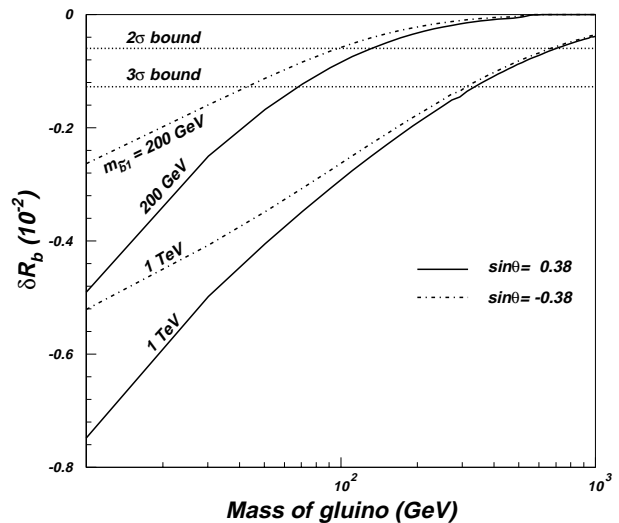


FIG. 3. Same as Fig. 2, but versus gluino mass for  $m_{\tilde{b}_2} = 3.5$  GeV.

Since in such a scenario with a light  $\tilde{b}_2$  of a few GeV, the  $\tilde{b}_1$  lighter than 200 GeV is disfavored by LEP 2 experiment [4], we fix  $m_{\tilde{b}_1} = 200$  GeV and  $\sin \theta = -0.3$  and plot  $\delta R_b$  versus  $m_{\tilde{b}_2}$  in Fig. 4, where  $m_{\tilde{b}_2}$  varies in the range  $2 \sim 5.5$  and  $m_{\tilde{g}}$  in  $12 \sim 16$  GeV, as used in [5] to explain the excess of bottom quark production in hadron collision. We see that such a scenario is totally excluded by the LEP  $R_b$  data at  $2\sigma$  level, while at  $3\sigma$  level only a tiny corner with  $m_{\tilde{g}}$  close to 16 GeV and  $m_{\tilde{b}_2}$  close to 5.5 GeV is allowed.

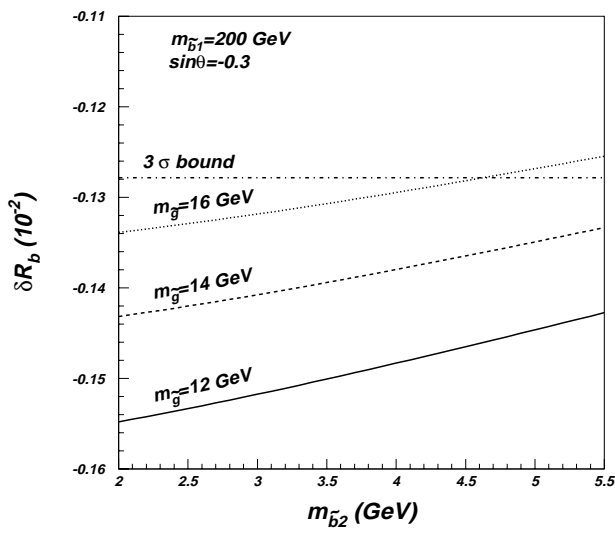


FIG. 4. Same as Fig. 2, but versus  $m_{\tilde{b}_2}$  for  $m_{\tilde{b}_1} = 200$  GeV.

Let's next consider the effects on other  $Z$ -peak observables:  $R_c$ ,  $R_\ell$ ,  $A_b$  and  $A_{FB}^b$ . In our calculation of these observables, we neglect SUSY QCD correction to  $\Gamma(Z \rightarrow q\bar{q})(q \neq b)$  since the corresponding loops involve heavy squarks  $\tilde{q}$  ( $\tilde{q} \neq \tilde{b}$ ) which are assumed to be heavy. Then the effects on all these observables stem only from the corrections to  $Zb\bar{b}$  vertex in eq.(5)<sup>2</sup>. In Table 1, we show the effects on these observables including  $R_b$ . We see that gluino-sbottom loop effects significantly enlarge the deviations of the predictions from the experimental values.

Table 1: Deviation of some  $Z$ -peak observables from experimental values. The MSSM predictions are obtained by including SUSY QCD contributions with  $m_{\tilde{b}_2} = 3.5$  GeV and  $m_{\tilde{g}} = 14$  GeV. The SM predictions are taken from [11]. The values of  $m_{\tilde{b}_1}$  are in units of GeV.

	MSSM						SM
	$\sin \theta = -0.30$			$\sin \theta = -0.45$			
	$m_{\tilde{b}_1}:$ 150	200	250	150	200	250	
$R_b$	$2.66\sigma$	$3.22\sigma$	$3.59\sigma$	$4.49\sigma$	$5.47\sigma$	$6.16\sigma$	$1.12\sigma$
$R_c$	$-0.19\sigma$	$-0.22\sigma$	$-0.24\sigma$	$-0.28\sigma$	$-0.33\sigma$	$-0.36\sigma$	$-0.12\sigma$
$R_\ell$	$2.26\sigma$	$2.66\sigma$	$2.93\sigma$	$3.65\sigma$	$4.32\sigma$	$4.83\sigma$	$1.11\sigma$
$A_b$	$-0.90\sigma$	$-0.93\sigma$	$-0.94\sigma$	$-0.76\sigma$	$-0.80\sigma$	$-0.84\sigma$	$-0.64\sigma$
$A_{FB}^b$	$-3.25\sigma$	$-3.28\sigma$	$-3.30\sigma$	$-3.06\sigma$	$-3.12\sigma$	$-3.16\sigma$	$-2.90\sigma$

At the end of this section, we should remind that in our calculation of SUSY corrections to  $Zb\bar{b}$  vertex, we only considered the SUSY QCD loops, i.e., gluino-sbottom loops. Since we focused on a special scenario of the MSSM, in which there exist a very light sbottom

( $2 \sim 5.5$  GeV) and a very light gluino ( $12 \sim 16$  GeV), such gluino-sbottom loop effects are much larger than SUSY electroweak corrections [7]. Therefore, although SUSY electroweak corrections could have the opposite sign to SUSY QCD corrections, the cancellation effect is very small and our conclusion remain valid.

**Conclusions** From the above analyses we conclude that the scenario of the MSSM with one light sbottom ( $2 \sim 5.5$  GeV) and light gluino ( $12 \sim 16$  GeV) can give rise to large effects on  $Zb\bar{b}$  vertex through gluino-sbottom loops. Such effects significantly enlarge the deviations of some  $Z$ -peak observables, especially  $R_b$ , from their experimental data. To account for the  $R_b$  data in this scenario, the other mass eigenstate of sbottom, i.e., the heavier one, must be lighter than 125 (195) GeV at  $2\sigma$  ( $3\sigma$ ) level, which, however, is disfavored by LEP II experiments.

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<sup>2</sup>Since  $F_{1,2}$  are found to be much larger than  $F_{3,4}$ , we neglect  $F_{3,4}$  in the calculation of  $A_b$  and  $A_{FB}^b$ .

